

***Abstract Interpretation for
Automatic Differentiation,
Runtime Error detection and
Security Analysis***

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AG60

Contents

- **One framework: abstract interpretation of program**
- **Three applications:**
 - Automatic Differentiation
 - Runtime Error detection
 - Automated Security analysis
- **Conclusion**
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Abstract interpretation: practical view

“Executing” programs according to a particular semantic

- Concrete value → abstract value**
- Concrete execution → abstract execution**

So that the abstract execution gives correct information about all possible concrete executions

Abstract interpretation: theoretical view

Way of computing

upper (resp. **lower**) approximations

of the

least (resp. **greatest**) fixpoint

of a monotonic function

from a complete lattice to itself

Theoretical framework

- **Galois connection: concrete \leftrightarrow abstract**
- **Abstract domains**
 - Finite / infinite
 - Relational / non relational
- **Abstraction**
 - Value \rightarrow set of values
 - Loop in the control \rightarrow Fixpoint iteration
- **Approximation**
 - Sources: Loop, compound object, union
 - Solutions: widening and narrowing operators

Example: Manual evaluation

$x := 2;$ $x = 2$

$y := 20;$ $y = 20$

while ($x < y$) $2 < 20, 4 < 19, 8 < 18, 16 < 17, 32 < 16$

{ $x := 2 * x;$ $x = 4, 8, 16, 32$

$y := y - 1;}$ $y = 19, 18, 17, 16$

Example: Abstract interpretation with Sign

Nearly no information

$x := 2;$ $x > 0$

$y := 20;$ $y > 0$

while ($x < y$)

{ $x := 2 * x;$ $x > 0$

$y := y - 1;$ } $y \rightarrow T$

Example: Abstract interpretation with interval

$x := 2;$ $x \in [2, 2]$

$y := 20;$ $y \in [20, 20]$

while ($x < y$) $[2, 2] < [20, 20]$ $[2, 4] < [19, 20]$ $[2, 8] < [18, 20]$
 $[2, 16] < [17, 20]$ $[2, 32] < [16, 20]$

{ $x := 2 * x;$ $x \in [4, 4]$ $[4, 8]$ $[4, 16]$ $[4, 32]$

$y := y - 1;$ } $y \in [19, 19]$ $[18, 19]$ $[17, 19]$ $[16, 19]$

$x \in [4, +\infty[$ $x \in 2\mathbb{Z} \cap [4..32]$

$y \in]-\infty, 19]$ $y \in [16..19]$

Automatic Differentiation

- **Static analysis**
 - Information
 - Variable aliases
 - Variable activity
 - depend on active inputs
 - Impact active outputs
 - Variable « to be stored » status
 - Modified
 - Overwritten
 - Very simple abstract domain
- **Code transformation**
 - Over approximation => generated program time / memory consuming
 - Static / dynamic => automatic generation of operator overloading
- **Tools: Odyssee -> Tapenade**

Runtime Error detection

- **Information**
 - Alias analysis
 - Value analysis

- **Property check**

$x := 2;$ $x \in [2, 2]$

$y := 20;$ $y \in [20, 20]$

while ($x < y$)

{ $x := 2 * x;$ $x \in [4, 32]$

$y := y - 1$ **};** $y \in [16, 19]$

$(x \neq 0 \ \&\& \ y \neq 0) = \text{True}$

$z := 1 / (x * y);$ **Division by zero**

Runtime Error detection (2)

- **Complex abstract domain**
 - Intervals
 - Congruencies
 - Polyhedrons
- **Static analysis**
 - Over approximation => properties (checks) not “proven”
 - Static / dynamic => automatic generation of unproven checks
- **PolySpace, Astrée, Frama C, Penjili, Fluctuat (numerical errors)**

Software security

- **Actual state**
 - > 200 static or static/dynamic tools
 - Find flaws/bugs/vulnerabilities in piece of software
 - Security analysis
 - Manual
 - Highly skilled people
- **Objective**
 - Computer aided software security analysis
 - Evaluating intrinsic exploitability of flaws
 - Potential attacks
 - Accessibility from inputs
 - Impact on output
 - ...
 - Evaluate effectiveness of protections

Conclusions

- **Static analysis**
 - Same problems (approximations => sometimes useless results)
 - Combine static / dynamic tools
- **Software security**
 - New field
 - Not yet structured (vocabulary / objectives / methods)
 - Exciting subject !

Andreas

- **We met at Santa Fe with Nicole Rostaing (1996)**
- **Gave me a flavor of AD**
 - Interesting / open field
 - Established community
- **Came at INRIA for one sabbatical year**
 - Lot of discussions on forward / backward ...
 - Merge static and dynamic AD
 - Checkpointing
 - Graphical description
 - First AD book
- **Nice AD Workshop (2000)**
- **AD project at INRIA**

Andreas (2)

- **I remember**
 - Apples and pieces of bread kept for later on
 - Questions about french grammar
 - Why do you say "this" and not "that" ?
 - What do you use « le présent du subjonctif » for ?
 - I could not answer !!!!
 - Discovery of the Piggy-Back
 - Industrial experiment with Odysée on Alenia code
 - One observation on a gnuplot figure of derivatives for me
 - A whole theory for him
 - Convergence acceleration of fix point iteration
 - Piggy-back optimisation